

```
In[1]:= Needs["IdentifiabilityAnalysis`"]
      startTime = AbsoluteTime[]
```

```
Out[2]= 3.6879579109293680 × 109
```

```
In[3]:= vars = {x1, x2, x3, x4}
```

```
Out[3]= {x1, x2, x3, x4}
```

```
In[4]:= params = {p1, p2, p3, p4, p5, p6, p7, p8, p9}
```

```
Out[4]= {p1, p2, p3, p4, p5, p6, p7, p8, p9}
```

```
In[5]:= sys = {x1'[t] == -p4 * x1[t] + p1 / (p2 + x4[t]),
               x2'[t] == p5 * x1[t] - p6 * x2[t],
               x3'[t] == p7 * x2[t] - p8 * x3[t],
               x4'[t] == (p3 (p7 x2[t] - p8 x3[t]) x4[t]) /
                           x3[t],
               x1[0] == 0.3, x2[0] == 0.9, x3[0] == 1.3, x4[0] == p9}
```

```
Out[5]= {x1'[t] == -p4 x1[t] + (p1 / (p2 + x4[t])), x2'[t] == p5 x1[t] - p6 x2[t],
         x3'[t] == p7 x2[t] - p8 x3[t], x4'[t] == (p3 (p7 x2[t] - p8 x3[t]) x4[t]) /
                                                    x3[t],
         x1[0] == 0.3, x2[0] == 0.9, x3[0] == 1.3, x4[0] == p9}
```

The original system can be rewritten by introducing an additional state variable  $z(t) = x_3(t)^{p_3}$  which implies the relations  $z'(t) = p_3 z(t) / x_3(t) x_3'(t) = p_3 z(t) / x_3(t) (p_7 x_2(t) - p_8 x_3(t))$  and  $z(0) = x_3(0)^{p_3} = 1.3^{p_3}$ . The last expression is further rewritten as  $z(0) = q$ , where  $q$  is a new parameter. Hence,

Now, from an input-output perspective the extended system in `sys` is equivalent to the original one except for the relation  $1.3^{p_3} = q$ , which is an invertible mapping, i.e., there is a one-one relation correspondence between  $p_3$  and  $q$ .

The extended list of state variables

The extended list of parameters

and the Identifiability Analysis of the extended system (except the non-rational relation  $1.3^{p_3} = q$ )

```
In[6]:= iad = IdentifiabilityAnalysis[{sys, output}, vars, params, t]
```

```
Out[6]= IdentifiabilityAnalysisData[False, <>]
```

The analyzed system is non-identifiable

```
In[7]:= iad["IdentifiableQ"]
```

```
Out[7]= False
```

with one degree of freedom, i.e., by fixing one of the non-identified parameters to a numerical value the non-identifiability will be resolved.

```
In[8]:= iad["DegreesOfFreedom"]
```

```
Out[8]= 9
```

The non-identifiable parameters are

```
In[9]:= iad["NonIdentifiableParameters"]
```

```
Out[9]= {p1, p2, p3, p4, p5, p6, p7, p8, p9}
```

We note that  $p_3$  is not among these, which means that by using the non-included relation  $1.3^{p_3} = q$  the value of  $q$  can be fixed and the original (and equivalent extended system) can be rendered identifiable.

```
In[10]:= endTime = AbsoluteTime[]
```

```
Out[10]= 3.6879579114615571 × 109
```

```
In[11]:= N[endTime - startTime]
```

```
Out[11]= 0.532189
```