Supplementary Material

**Evaluating Complexity of Fetal MEG Signals: A Comparison of Different Metrics and Their Applicability**

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# Scale-free approaches

A given process is said statistically self-similar if its statistical properties are invariant after rescaling and time dilating

(1)

where denotes statistical equivalence and is a scalar scaling factor. This self-similarity implies a power-law behavior of statistical moments so that the *q*th statistical moment of the processis expressed as

(2)

where *H* refers to the *Hurst* exponent. In the multifractality scheme, is no longer characterized by one exponent, but rather several exponents *h*, called *Hölder* exponents, forming the multifractality spectrum whose maximum coincides with the *Hurst* exponent. The *q*th statistical moment is then expressed as

(3)

where is a scaling polynomial (concave) function. The latter is related to the multifractality spectrum (a.k.a. singularity spectrum) via the Legendre transform

(4)

Multifractal analysis amounts to analyze the signal across different scales .There are two practical well-known approaches to measure fractality in a process (if it exists).

## Multifractal detrended fluctuation analysis

Given a time series, the fluctuating function is defined as

= (5)

where *s* and *N* represent, the scale and the number of segments of length, respectively. is the standard deviation of the detrended signal in segment *i* at scale *s*. When is multifractal, the fluctuating function presents a power-law scaling behavior

(6)

where *H*(*q*) denotes the generalized *Hurst* exponent (*q*-dependent). The latter is related to the scaling function by and to the *Hölder* exponent by. The Multifractal detrended fluctuation analysis (MFDFA) computes by linearly regressing versus the scale log(*s*) for each value of *q*. After few computational steps (detailed in (Kantelhardt 2002)), the multifractal spectrum can be calculated, without using the Legendre transform, as

(7)

The *Hurst* exponent *H* and the spectral width *M* correspond to the *h* maximizing and the width of, respectively.

In the monofractal case, is reduced to a linear function of *q* so that, where *H* is the *Hurst* exponent.

## Wavelet Leader-based multifractal approach

Given a time series and for a fixed analysis scale, the structure function is defined as

(8)

where is the wavelet leaders coefficient at scale *j* and time *k* and is the number of available at scale . When is a fractal process, the structure function shows a power-law behavior

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By noting that is a sample mean estimator of and using the standard generating function expansion, the following relation can be established

(10)

where stands for the cumulants of of order . Combining (9) and (10) compels that these cumulants satisfy the following form

= (11)

which consequently yields. The characterization of (consequently) amounts to calculating the log-cumulants. In terms of interpretations, the log-cumulants and characterize the maximum and width of the multifractal spectrum. In the WLBMF method, the log-cumulants are estimated by linearly regressing the estimate of cumulants versus in the analyzed scales range

(12)

The estimates are calculated using the standard methods of cumulant estimators. For more details, see (Wendt, Abry et al. 2007).

**References**

Kantelhardt, J. W. Z., Stephan A.; Koscielny-Bunde, Eva; Havlin, Shlomo; Bunde, Armin; Stanley, H. Eugene (2002). "Multifractal Detrended Fluctuation Analysis of Nonstationary Time Series." Physica A: Statistical Mechanics and its Applications **316**(1): 87-114.

Wendt, H., P. Abry and S. Jaffard (2007). "Bootstrap for Empirical Multifractal Analysis." IEEE Signal Processing Magazine **24**(4): 38-48.