Contact Tracing – Old Models and New Challenges – SI

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1 Contact Tracing – Literature overview

The table orders the papers primarily according to the infection they aim to investigate resp. for the rather theoretical papers, by the method used. If several infections are discussed, the paper nevertheless appears only once.

Abbreviations:
CT: contact tracing
DCT: digital contact tracing
phenomen.: phenomenological approach (ODE)

author	infection	method	term for CT	CT delay	remarks	outcome(w.r.t. CT)
	Chlamydia & Gonorrhoea					
Clark et al. 2012 [20]	Chlamydia	pair approx.	removal rate on infected-diagnosed pairs	-	based on [26, 40]	CT is efficient particularly below a certain prevalence
Clark et al. 2013 [21]	Chlamydia	IBM + phenomen. ODE SI	IBM: direct; ODE: power law in I	-	power law adapted to IBM simulations; optimal resource allocations (CT/screening)	CT is efficient and uses resources efficient
Heffernan et al. 2009 [36]	Chlamydia	phenomen. / ODE SI	mass action law	-	model includes random screening (yield index cases) and CT	model results in line with data
Hethcote et al 1984 [39]	Gonorrhoea	phenomen. ODE	decreased incidence	-	introduce backward and forward tracing	pioneering work about CT
Kretzschmar et al. 1996 [57]	Gonorrhoea, Chlamydia	IBM with household structure	identification of a fraction p of partners	-	one-step tracing	prevalence for different control scenarios
Kretzschmar et al. 2009 [55]	Chlamydia	IBM	direct formulation		three different IBM models previously published by different authors are compared	the results of the models are somewhat different, due to their complexity
Turner et al. 2006 [83]	Chlamydia	IBM	direct formulation	+	model with pair formation, CT within pairs	Model fits data, and yields comparable results comparable studies
	Ebola					
Berge et al. 2018 [13]	Ebola	phenomen. / ODE, SEIR	fixed fraction of newly infecteds go to quarantine	-	CT not explicitly formulated	stationary states and their stability analysed
Browne et al. 2015 [17]	Ebola	branching process/ODE, SEIR	fixed fraction of newly infecteds go to quarantine	+	fraction of detected cases is computed based on the branching-process	paper aims at a theoretical framework that is feasible for practical applications
Shahtori et al. 2018 [78]	Ebola	IBM, homogeneous population	direct formulation	+	onset of infection only	crucial that CT is implemented at the beginning of the outbreak
Rivers et al. 2014 [76]	Ebola	phenomen., stoch. sim on pop.-level.+ODE, SEIR	increased diagnosis rate	-	CT not explicitly formulated	reduction of R_{eff} by around 30% possible

author	infection	method	term for CT	CT delay	remarks	outcome(w.r.t. CT)
	HIV					
de Arazoza et al. 2002 [23]	HIV	phenomen. / ODE SI	mass action	-	three classes of infecteds modelled: infecteds do /do not know their infection, AIDS	stability analysis of stationary points, comparison with data
Clémençon et al. 2008 [22]	HIV	phenomen. / stoch. process (population level), SDE, ODE	several terms: linear, mass action, saturation function	-	extension of de Azoza [23]	development of statistical tools (maximum likelihood estimators)
Blum et al. 2010 [14]	HIV	phenomen. / stochastic model on population level	linear and saturation	+	see also [68, 79]	Bayes interference (ABC and Metropolis Hastings) for estimating ct probability
Hsieh et al. 2005 [41]	HIV	phenomen. / ODE SI	several terms: linear, mass action, saturation function	-	extension of de Azoza [23]	mass action inappropriate, linear or saturation term for CT better
Hsieh et al. 2010 [42]	HIV	phenomen. / ODE SI	saturation function; two-step tracing	-	extension of de Azoza [23]	stability analysis of stationary points, R_{eff}; two-step tracing superior over one-step tracing
Hyman et al. 2003 [44]	HIV	phenomen. / ODE SI	mass action	-	two models: core group, different stages of HIV	R_{eff}, sensitivity analysis
Mellor et al. 2001 [66]	HIV and Tuberculosis	IBM with household structure	screening the household	-	casual contacts are not traced; HIV and Tuberculosis at the same time	cross-tracing of HIV and Tuberculosis is effective
Naresh et al. 2006 [72]	HIV	phenomen. / ODE SI	fixed fraction of newly infecteds know their infection	-	CT is not triggered by diagnosis, but infections of known infecteds	stability analysis of stationary points
	Influenza					
Agarwal et al. 2012 [1]	Influenza	phenomen. / ODE SIR	fraction of newly infecteds go to quarantine	-	two risk classes in susceptibles are considered	Dynamical systems analysis, R_{eff}
	Measles					
Liu et al. 2015 [62]	Measles	IBM, inhomogeneous population	direct formulation	+	complex/realistic contact structure	CT can significantly contribute to control a measles outbreak

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	SARS					
Becker et al. 2005 [11]	SARS	phenomen. / next generation operator	reduction of incidence	-	article exclusively aims at R_{eff}, no dynamics.	formula for R_{eff}; social distancing together with CT can control SARS
Chen et al. 2006 [19]	Influenza, Measles, Chickenpox,SARS	phenomen. / PDE	fixed fraction of newly infecteds are eventually traced	-	based on Fraser [33]; only R_{eff}, no dynamics; airborne infection	Probability to control an outbreak is estimated
Kwok et al. 2019 [61]	SARS				review article	
Fraser et al. 2004 [33]	Theory & Influenza, SARS, Smallpox, HIV	phenomen. / PDE	fixed fraction of newly infecteds are eventually traced	-	basic model, used in applications [19, 31]	timing of incubation period and latency period is central for CT
Lloyd-Smith et al. 2003 [63]	SARS	phenomen. / time-discrete stoch. sim on pop.-level	increased transition rate to quarantine	+	CT not explicitly formulated	crucial that CT is implemented at the beginning of the outbreak
	SARS-CoV-2					
Aleta et al. 2020 [2]	SRAS-CoV-2	IBM, inhomogeneous population	direct formulation	+	delailed social graph for the Boston area used	a second infection wave can be controlled by massive testing/tracing; CT is indispensable.
Barrat et al. 2020 [9]	SARS-CoV-2	IBM, inhomogeneous population	direct formulation	+	follow-up of Di Dimenico et al. 2020 [24] with DCT; preprint	Linear effect for manual CT in tracing probability, quadratic effect for DCT
Bradshaw et al. 2020 [15]	SARS-CoV-2	IBM, homogeneous population	direct formulation	+	preprint; DCT considered, focus on onset, R_{eff} and prob. for major outbreak	backward tracing and high abundance of DCT devices necessary
Braithwaite et al. 2020 [16]	SARS-CoV-2				review article, focused on DCT	
Bulchandani et al. 2020 [18]	SARS-CoV-2	IBM, homogeneous population	direct formulation	-	preprint; DCT considered, focus on onset, heuristic formula for R_{eff}	High DCT-device coverage necessary
Di Dimenico et al. 2020 [24]	SRAS-Cov-2	IBM, inhomogeneous population	direct formulation	+	age-structured IBM with social graph for Île-de-France; preprint	CT necessary for control; lifting the lockdown with no exit strategy induces a huge second wave
Ferretti et al. 2020 [31]	SARS-CoV-2	phenomen. / PDE	fixed fraction of newly infecteds are eventually traced	+	based on Fraser [33]; DCT considered, heuristic formula for R_{eff}	SARS-CoV-2 controllable by DCT
Firth et al. 2020 [32]	SRAS-CoV-2	IBM, inhomogeneous population	direct formulation	+	DCT with informing contactees of contactees (second level tracing)	tracing+testing can prevent a second wave, a high fraction of persons need to be quarantined
Giordano et al. 2020 [34]	SARS-CoV-2	phenomenol., ODE/SIR	increased rate for removal	-	detailed ODE model	CT is a central element in controlling the infection

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Grasly et al. 2020 [35]	SARS-CoV-2	Phen. approach	approximates linearly one-step tracing in a tree	-	resembles approach by Fraser [33]	R_{eff} approximated; testing, tracing, quarantine and social distancing can control covid-19
Hellewell et al. 2020 [37]	SARS-CoV-2	IBM, homogeneous population	direct formulation	-	onset of epidemic considered	high tracing probability necessary to control the infection
Hernandez-Orallo et al. 2020 [38]	SARS-CoV-2	IBM, inhomogeneous population + ODE	direct formulation (IBM) / fixed fraction of newly infecteds are eventually traced (ODE)	+	simulations based on empirical contact network	CT needs to be precise in order to avoid many persons in quarantine
Juneau et al. 2020 [45]	SRAS-CoV-2				review article; preprint	
Keeling et al. 2020 [49]	SARS-CoV-2	IBM, homogeneous population	direct formulation	-	preprint; time and intensity of contacts vary, simulation-based estimation of R_{eff};	high tracing probability necessary to control the infection
Kim et al. 2020 [50]	SARS-CoV-2	Stochastic branching process	verbally	-	preprint; DCT considered; Heuristic calculations of efficiency	heuristic formula for efficiency
Kretzschmar et al. [59]	SARS-CoV-2	IBM, homogeneous population	direct formulation	+	preprint; Model based on Kretzschmar(2004) [56]; preprint	middle range tracing probability necessary to control diseases
Kretzschmar et al. [58]	SARS-CoV-2	IBM, homogeneous population	direct formulation	+	preprint; Model based on Kretzschmar [56]; DCT considered, combined with conventional CT; preprint	tracing delays need to be minimised for effective CT; DCT might be a way to speed up the process
Kucharski et al. [60]	SARS-CoV-2	IBM, inhomogeneous population	direct formulation	-	preprint; DCT considered, combined with conventional CT	CT more efficient than mass testing
Lorch et al. [64]	SARS-CoV-2	IBM with discrete spatial structure	one-step tracing, if at similar times in the same location	-	DCT considered, combined with conventional CT; preprint	DCT efficient particularly in case of a low fraction of quarantined persons

author	infection	method	term for CT	CT delay	remarks	outcome (w.r.t. CT)
Lunz et al. [65]	SARS-CoV-2	phenomen. / ODE SEIR	removal rate, mass action	-	the tracing rate is connected to contact heterogeneity but not based on first principles; preprint	optimal CT defined as minimising the total number of individuals that go into quarantine during he outbreak
Pollmann et al. 2020 [74]	SARS-CoV-2	IBM, (homogeneous and inhomogeneous version) and two branching process models	direct formulation	+	IBMs with models of type [71] and [17] compared; analytic results for DCT in branching models; preprint	CT is able to control the outbreaks, many individuals go to quarantine without precise protocol
Tanaka et al. [79]	SARS-CoV-2	IBM, homogeneous population	direct formulation	-	simulation of clusters detected by CT as input for stats (see also Blum [14]); preprint	Bayesian parameter estimation based on CT
	Smallpox					
Eichner 2003 [28]	Smallpox	phenomen. / Stochastic model on population level	all close contacts and fraction of casual contacts are traced	-	age of infection included in the model	critical tracing probability estimated
Kaplan et al. 2002 [46]	Smallpox	phenomen. / ODE	saturation function	-	contactees who are traced are vaccinated	mass vaccination superior to vaccination triggered by CT
Kretzschmar et al. [56]	Smallpox	Stochastic branching process	direct formulation	+	ring vaccination triggered by contact tracing	delay in CT is crucial
Porco et al. 2004 [75]	Smallpox	IBM with household structure	direct formulation	+	one step and two-step tracing compared	massive CT and ring vaccination can control the outbreak.
	Tuberculosis					
Aparicio et al. 2006 [3]	Tuberculosis	phenomen. / ODE SEIR	fixed fraction of newly infecteds are identified	-	CT not explicitly formulated	simulation of prevalence
Begun et al. 2013 [12]	Tuberculosis				review article	
Kasaie et al. 2014 [47]	Tuberculosis	IBM with household structure	screening the household	-	contacts outside the household are not traced	household tracing reduces the incidence by 2%-3%
Tian et al. 2011 [80]	Tuberculosis	IBM, inhomogeneous population	direct formulation	+	different scenarios, sensitivity analysis	simulation of prevalence
Tian et al. 2013 [81]	Tuberculosis	IBM, inhomogeneous population	direct formulation	+	Follow-up of Tien [80]	simulation of prevalence

author	infection	method	term for CT	CT delay	remarks	outcome(w.r.t. CT)
	Theory	**branching process**				
Ball et al. 2011 [6]	Theory	Stochastic branching process	direct formulation	-	SIR, focus on fixed and exponentially distributed infectious period	analytic approach, bounds on R_{eff}, extinction probability
Ball et al. 2015 [7]	Theory	Stochastic branching process	direct formulation	+	SEIR, follow up of Ball [6]	effect of tracing delay, R_{eff}, extinction prob.
Barlow 2020 [8]	Theory	Branching process	direct formulation	-	extends the perculation-based analysis/generating functions to CT; preprint	approximate expression for the probability for extinction
Baumgarten et al. 2020 [10]	Theory	Stochastic branching process	direct formulation	-	uses ideas of percolation theory; time-discrete model; DCT; preprint	Approximation of R_0, compares result with stochastic simulations
Endo et al. 2020 [29]	Theory	Stochastic branching process	direct formulation	-	uses three generations to approximate the effect of CT with an index case in the middle generation (backward+forward tracing); time-discrete process; preprint	approximate results for R_{eff} in presence of overdispersion
Müller et al. 2000 [71]	Theory	Stochastic branching process	direct formulation	-	focus on age since infection	R_{eff}, ODE approximation, critical tracing probability
Müller et al. 2007 [68]	Theory	Stochastic branching process	direct formulation	-	based on Müller [71], see also [14, 79]	estimation of tracing probability
Müller et al. 2016 [70],	Theory	Stochastic branching process	direct formulation	+	based on Müller [71]	effect of tracing delay and latency period, R_{eff}
Müller et al. 2020 [69],	Theory	Stochastic branching process	direct formulation	-	based on Müller [71], focus on superspreader events	Mechnism fo CT in case of superspreaders is different from usual CT
Okolie et al. 2018 [73]	Theory	Stochastic branching process	direct formulation	-	connects branching process and pair approx., based on Müller [71]	effect of a random contact graph on CT
Klinkenberg et al. 2006 [53]	Theory & influenza, smallpox, SARS, foot-and-mouth disease	Stochastic branching process	direct formulation	+	single-step and recursive tracing	mostly: single step and recursive tracing is equal effective
Shaban et al. 2008 [77]	Theory	Stochastic branching process	direct formulation	-	Vaccination of detected individuals	R_{eff}, probability for extinction, simulation of final size
Kojaku et al. 2020 [54]	Theory	Stochastic branching process	direct formulation	-	CT on random graph, focus on generating functions for the degree; preprint	CT highly efficient as nodes with high degree are traced

author	infection	method	term for CT	CT delay	remarks	outcome(w.r.t. CT)
	Theory	**IBM**				
Armbruster et al. 2007 [4]	Theory	IBM, lattice graph	direct formulation	-	addresses a cost-efficiency analysis; follow-up of Armbruster et al. [5]	CT only cost efficient if prevalence is small
Kiss et al. 2006 [51]	Theory	IBM, inhomogeneous population	direct formulation	-	isolation of susceptible; scale free and Poisson network	For scale free networks, tracing effect less sensitive to the epidemiological parameters
Kiss et al. 2008 [52]	Theory	IBM, inhomogeneous population	direct formulation	-	assortatively / disassortatively mixing networks; Single-step and recursive tracing	CT more effective in dissacociative networks; recursive tracing more efficient
Farrahi et al. [30]	Theory	IBM, inhomogeneous population	direct formulation	+	first model for DCT	DCT can be efficient even if the fraction of app-users is small
	Theory	**pair approximation**				
Eames et al. 2002 [26]	Theory / STD	pair approx.& stoch sim.	direct formulation	-	this paper introduced pair approximation for CT	modelling CT by pair approximation
Eames et al. 2003 [27]	Theory	pair approx.& stoch sim.	direct formulation	-	based on Eames [26]	critical tracing probability
Eames et al. 2005 [48]	Theory	pair approx.& stoch sim.	direct formulation	-	based on [26]; focus on different social graphs (small world, scale-free)	network structure influence efficiency of CT
Eames 2007 [25]	Theory	pair approx.& stoch sim.	direct formulation	-	based on Eames [26]	recursive CT much more effective than one-step CT; "targeted CT": focus on core groups
House et al. 2010 [40]	Theory	pair approx.& stoch sim.	direct formulation	-	based on Eames [26]; focus on different social graphs (small world, scale-free)	CT has higher efficiency in clustered contact graphs
Huerta et al. 2002 [43]	Theory	pair approx.& stoch sim.	direct formulation	-	develop pair approximation for CT	rewiring of contact network decreases CT
Tsimering et al. 2003 [82]	Theory	pair approx.& stoch sim.	direct formulation	-	based on Huerta [43]	rewiring of contact network decreases CT
	Theory	**phenomenological approach**				
Armbruster et al. 2007 [5]	Theory	phenomen. / ODE	linear removal term	-	addresses a cost-efficiency analysis	CT only cost efficient if prevalence is small
Mizumoto et al. 2013 [67]	Theory	phenomen. / next generation operator	reduction of R0 by a factor	-	focus on the onset; multitype-branching process, analysed by generating functions	R_{eff}, probability for extinction, duration of a minor outbreak

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