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THE PEŁCZYŃSKI AND DUNFORD–PETTIS PROPERTIES OF THE SPACE OF UNIFORM CONVERGENT FOURIER SERIES WITH RESPECT TO ORTHOGONAL POLYNOMIALS

BY

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Abstract. The Banach space $U(\mu)$ of uniformly convergent Fourier series with respect to an orthonormal polynomial sequence with orthogonalization measure μ supported on a compact set $S \subset \mathbb{R}$ is studied. For certain measures μ, involving Bernstein–Szegö polynomials and certain Jacobi polynomials, it is proven that $U(\mu)$ has the Pełczyński property, and also that $U(\mu)$ and $U(\mu)^\star$ have the Dunford–Pettis property.

1. Introduction. Let $S \subset \mathbb{R}$ be a compact infinite set and μ be a Borel measure on $\mathbb{R}$ with $\mu(\mathbb{R}) = 1$ and $\operatorname{supp} \mu = S$. By Gram–Schmidt procedure there exists a unique sequence $\{p_n\}_{n=0}^\infty$ of algebraic polynomials such that $\int p_n p_m \, d\mu = \delta_{n,m}$, $\deg p_n = n$ and p_n has a positive leading coefficient. We call $\{p_n\}_{n=0}^\infty$ the *orthonormal polynomial sequence with respect to* μ. In particular, we have $p_0 \equiv 1$. It is well-known [2] that there holds a three-term recurrence relation

$$x p_n(x) = \lambda_{n+1} p_{n+1}(x) + \beta_n p_n(x) + \lambda_n p_{n-1}(x) \quad \text{for all } n \in \mathbb{N}_0, \tag{1.1}$$

where $p_{-1} \equiv 0$, $\{\lambda_n\}_{n=0}^\infty, \{\beta_n\}_{n=0}^\infty \subset \mathbb{R}$ are bounded sequences, $\lambda_n > 0$ for all $n \in \mathbb{N}$ and λ_0 is arbitrary.

As usual, let

$$C(S) = \{f : S \to \mathbb{C} : f \text{ continuous}\} \tag{1.2}$$

with norm $\|f\|_\infty = \sup_{x \in S} |f(x)|$. The formal Fourier series of $f \in C(S)$ is given by

$$f \sim \sum_{n=0}^\infty \hat{f}_n p_n, \tag{1.3}$$

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where the Fourier coefficients are defined by

$$\hat{f}_n = \int f p_n \, d\mu. \tag{1.4}$$

By the Weierstrass theorem the polynomials $\{p_n\}_{n=0}^{\infty}$ form a complete orthonormal system in $L^2(\mu)$, therefore the coefficients $\hat{f}_n$ determine the functions $f \in C(S)$.

The Nth partial sum of the formal Fourier series of $f \in C(S)$ is given by

$$D_N(f) = \sum_{n=0}^{N} \hat{f}_n p_n. \tag{1.5}$$

Let

$$U(S, \mu) = U(\mu) = \left\{ f \in C(S) : \lim_{N \to \infty} \|D_N(f) - f\|_\infty = 0 \right\} \tag{1.6}$$

denote the subspace of $C(S)$ with uniformly convergent Fourier series. Then

$$\|f\|_U = \sup_{N \in \mathbb{N}_0} \|D_N(f)\|_\infty \tag{1.7}$$

defines a norm on $U(\mu)$, which we call the *U-norm.* It is well-known that $(U(\mu), \| \,\|_U)$ is a Banach space (see [17, Proposition 3.1]). Obviously,

$$\|f\|_\infty \le \|f\|_U \quad \text{for all } f \in U(\mu). \tag{1.8}$$

If $S = [a, b]$, then $U(\mu) \subsetneq C(S)$ (see [4]). But there are discrete S such that $U(\mu) = C(S)$ (see [10, 11, 12, 13]). In such a case the open mapping theorem yields the equivalence of the norms, i.e. there exists $C > 0$ such that

$$C\|f\|_U \le \|f\|_\infty \le \|f\|_U \quad \text{for all } f \in C(S). \tag{1.9}$$

But in general, less is known about the Banach space $(U(\mu), \| \,\|_U)$. For investigating $U(\mu)$ later on, the following lemma is of fundamental importance. The proof is based on ideas of [16].

LEMMA 1.1. *Let $(U(\mu), \| \,\|_U)$ be a Banach space of uniformly convergent Fourier series with respect to an orthonormal polynomial sequence. Then there exists a compact Hausdorff space K and an isometry J from $U(\mu)$ into the Banach space $C(K)$ of complex-valued continuous functions on K.*

Proof. Since

$$S_d = \{1/n : n \in \mathbb{N}\} \cup \{0\}$$

with the topology induced by the topology of $\mathbb{R}$ is a compact Hausdorff space, so is

$$K = S_d \times S$$

with the product topology. For $\varphi \in C(K)$ set

$$f_{n-1}(x) = \varphi((1/n, x)), \qquad n \in \mathbb{N} \cup \{\infty\},\ x \in S.$$

Here we assume that $1/\infty = 0$ and $\infty - 1 = \infty$. Then $f_n \in C(S)$ for all $n \in \mathbb{N}_0 \cup \{\infty\}$ and f_n converges uniformly to f_∞ as $n \to \infty$. The other way round, if $f_n \in C(S)$ and $\{f_n\}_{n=0}^{\infty}$ converges uniformly to $f_\infty \in C(S)$ as $n \to \infty$, then $\varphi((1/n, x)) = f_{n-1}(x)$, $n \in \mathbb{N} \cup \{\infty\}$, $x \in S$, defines a function in $C(K)$. Thus $C(K)$ is isometrically isomorphic to the Banach space

$$C_{\mathrm{seq}} = \{\{f_n\}_{n=0}^{\infty} : f_n \in C(S) \text{ and } \{f_n\}_{n=0}^{\infty} \text{ is uniformly convergent}\},$$

where the norm on C_{seq} is given by $\|\{f_n\}_{n=0}^{\infty}\|_{C_{\mathrm{seq}}} = \sup_{n\in\mathbb{N}_0} \|f_n\|_\infty$. If we denote the isometry from C_{seq} onto $C(K)$ by J_2 and the isometry from $U(\mu)$ into C_{seq} by J_1, which is defined by $J_1(f) = \{D_n(f)\}_{n=0}^{\infty}$, then

$$J = J_2 \circ J_1$$

is an isometry from $U(\mu)$ into $C(K)$. ■

For the investigation of $U(\mu)$ we focus on boundedness properties of orthonormal polynomial sequences.

2. Boundedness properties of orthonormal polynomial sequences.

In this section we deal with measures μ which give rise to special boundedness properties of the sequence $\{p_n\}_{n=0}^{\infty}$.

DEFINITION 2.1. Let $\{p_n\}_{n=0}^{\infty}$ denote the orthonormal polynomial sequence with respect to μ. If $\{\|p_{n+1}\|_\infty/\|p_n\|_\infty\}_{n=0}^{\infty}$ and $\{\|p_n\|_\infty/\|p_{n+1}\|_\infty\}_{n=0}^{\infty}$ are bounded sequences, then we say μ has *property* (PB). If $\{\|p_n\|_\infty\}_{n=0}^{\infty}$ is bounded, then we say μ has *property* (B).

Notice that $\|p_n\|_\infty \geq 1$ for all $n \in \mathbb{N}_0$ and therefore property (B) implies property (PB). The following lemma is fundamental to the achievement of our goal. Let $\mathcal{P}$ denote the set of polynomials in one variable with complex coefficients.

LEMMA 2.2. *If μ has property* (PB), *then $\mathcal{P}U(\mu) = U(\mu)$.*

Proof. Let $f \in U(\mu)$ and let $c_n = \int f p_n \, d\mu$ denote the Fourier coefficients of f with respect to the orthonormal polynomial sequence $\{p_n\}_{n=0}^{\infty}$. By (1.1) the Fourier coefficients of $xf(x)$ are given by $d_n = c_{n-1}\lambda_n + c_n\beta_n + c_{n+1}\lambda_{n+1}$, and

$$\begin{aligned} x\sum_{n=0}^{N} c_n p_n(x) &= \sum_{n=0}^{N} c_n(\lambda_{n+1}p_{n+1}(x) + \beta_n p_n(x) + \lambda_n p_{n-1}(x)) \\ &= \sum_{n=1}^{N+1} c_{n-1}\lambda_n p_n(x) + \sum_{n=0}^{N} c_n\beta_n p_n(x) + \sum_{n=0}^{N-1} c_{n+1}\lambda_{n+1} p_n(x) \\ &= \sum_{n=0}^{N} d_n p_n(x) + \lambda_{N+1}(c_N p_{N+1}(x) - c_{N+1}p_N(x)). \end{aligned}$$

Thus

$$\Big\|xf(x)-\sum_{n=0}^{N} d_n p_n(x)\Big\|_\infty \le \sup_{x\in S}|x|\,\|f-D_N(f)\|_\infty+\lambda_{n+1}\|c_N p_{N+1}-c_{N+1}p_N\|_\infty$$

and

$$\begin{aligned}\lambda_{n+1}&\|c_N p_{N+1}-c_{N+1}p_N\|_\infty\\ &= \Big\|xD_N(f)(x)-\sum_{n=0}^{N} d_n p_n(x)\Big\|_\infty\\ &\le \sup_{x\in S}|x|\,\|D_N(f)-f\|_\infty+\Big\|xf(x)-\sum_{n=0}^{N} d_n p_n(x)\Big\|_\infty.\end{aligned}$$

Hence, the Fourier series of $xf(x)$ is uniformly convergent if and only if

$$\lim_{N\to\infty}\|c_N p_{N+1}-c_{N+1}p_N\|_\infty=0.$$

As $\lim_{N\to\infty}\|c_N p_N\|_\infty=0$, by property (PB) we get $\lim_{N\to\infty}\|c_N p_{N+1}\|_\infty=0$ and $\lim_{N\to\infty}\|c_{N+1}p_N\|_\infty=0$. ∎

Let us give some examples of measures μ with properties (PB) and (B).

Example 2.3. The Jacobi measure

$$d\mu^{(\alpha,\beta)}=\frac{\Gamma(\alpha+\beta+2)}{2^{\alpha+\beta+1}\Gamma(\alpha+1)\Gamma(\beta+1)}(1-x)^\alpha(1+x)^\beta dx,\qquad \alpha,\beta>-1,$$

is supported on $[-1,1]$. If $-1<\alpha,\beta\le -1/2$, then $\mu^{(\alpha,\beta)}$ has property (B), whereas in the case $\alpha>-1/2$ or $\beta>-1/2$ the Jacobi measure has only property (PB) but not (B) (see [18, Theorem 7.32.1]).

Example 2.4. Let $q\in\mathcal{P}$ with $q(x)>0$ for all $x\in[-1,1]$ and

$$C_q=\Big(\int_{-1}^{1}(1-x^2)^{-1/2}q(x)^{-1}\,dx\Big)^{-1}.$$

Then

$$d\mu_q(x)=C_q(1-x^2)^{-1/2}q(x)^{-1}dx$$

is called the *Bernstein–Szegö measure with respect to* q (see [18, (2.6.1)]). The Bernstein–Szegö measure has property (B) for any q (see [8]).

3. The Pełczyński property with respect to $U(\mu)$. For notations and definitions in this and in the next sections we also refer to [9]. Let X and Y be Banach spaces. A series $\sum_n x_n$ in X is called *weakly unconditionally convergent* if the series $\sum_n x^\star x_n$ is absolutely convergent for all $x^\star\in X^\star$, and a continuous linear operator $T:X\to Y$ is called *unconditionally convergent* if it maps weakly unconditionally convergent series to unconditionally convergent ones. An immediate consequence of a result of Orlicz [14] is that every weakly compact operator T is unconditionally convergent. The converse was introduced by Pełczyński [15] as property (V). In honor of Pełczyński the following definition was made.

DEFINITION 3.1. A Banach space X is said to have the *Pełczyński property* if every unconditionally convergent operator $T : X \to Y$ is weakly compact.

There are other characterizations of the Pełczyński property (see [16, Proposition 1.1]). For instance all reflexive Banach spaces and the Banach space $C(K)$ of continuous scalar-valued functions on a compact Hausdorff space K have the Pełczyński property [15]. Hence, if $U(\mu) = C(S)$, then $U(\mu)$ has the Pełczyński property.

The main result of Saccone [16, Theorem 2.1] is that so-called tight subspaces of continuous function spaces have the Pełczyński property.

DEFINITION 3.2. Let K be a compact Hausdorff space and $X \subset C(K)$ be a closed subspace of the space of continuous scalar-valued functions on K. Then X is said to be a *tight subspace* if the operator $T_\gamma : X \to C(K)/X$, $\varphi \mapsto \varphi\gamma + X$, is weakly compact for every γ in $C(K)$.

Now, the following theorem holds.

THEOREM 3.3. *If μ has property* (B)*, then $U(\mu)$ has the Pełczyński property.*

Proof. We are using the notations and definitions of the proofs of Lemmas 1.1 and 2.2. Set $X = J(U(\mu))$ and $X_{\mathrm{seq}} = J_1(U(\mu))$.

By [16, Theorem 2.1] it is sufficient to prove that X is a tight subspace of $C(K)$. For that purpose let

$$T_\gamma : X \to C(K)/X, \qquad \varphi \mapsto \varphi\gamma + X,$$

where $\gamma \in C(K)$ and

$$Y = \{\gamma \in C(K) : T_\gamma \text{ is weakly compact}\}.$$

It is known that Y is a closed subalgebra of $C(K)$ (see [16, p. 151]). Hence, by the Stone–Weierstrass theorem it remains to prove that there is a self-adjoint subset of Y which separates the points of K.

Let $\phi_k = \{\delta_{k,n}\}_{n=0}^\infty \in C_{\mathrm{seq}}$, $k \in \mathbb{N}_0$, and $\phi_\infty = \{x\}_{n=0}^\infty \in C_{\mathrm{seq}}$, where $0, 1$ in ϕ_k stand for the corresponding constant functions, and x in ϕ_∞ stands for the identity function on S. It is easy to show that $\{J_2(\phi_k) : k \in \mathbb{N}_0 \cup \{\infty\}\}$ is self-adjoint and separates the points of K.

Now, we may study the corresponding operators T_{ϕ_k} and T_{ϕ_∞} on X_{seq}. Notice that $f \in U(\mu)$ if and only if $\{D_n(f)\}_{n=0}^\infty \in X_{\mathrm{seq}}$.

For $k \in \mathbb{N}_0$ we have

$$T_{\phi_k}(\{D_n(f)\}_{n=0}^{\infty}) = \{\delta_{k,n} D_n(f)\}_{n=0}^{\infty} + X_{\mathrm{seq}}.$$

Hence, T_{ϕ_k} is of finite rank and therefore weakly compact.

If $k = \infty$, then

$$\begin{aligned} T_{\phi_\infty}(\{D_n(f)\}_{n=0}^{\infty}) &= \{x D_n(f)\}_{n=0}^{\infty} + X_{\mathrm{seq}} \\ &= \{\lambda_{n+1}(c_{n+1}p_n - c_n p_{n+1})\}_{n=0}^{\infty} + \{D_n(x\,f)\}_{n=0}^{\infty} + X_{\mathrm{seq}} \\ &= \{\lambda_{n+1}(c_{n+1}p_n - c_n p_{n+1})\}_{n=0}^{\infty} + X_{\mathrm{seq}} \end{aligned}$$

(see the proof of Lemma 2.2).

Since the canonical quotient map $C_{\mathrm{seq}} \to C_{\mathrm{seq}}/X_{\mathrm{seq}}$ is continuous and the composition of two continuous operators is weakly compact if one of them is [9, Proposition 3.5.11], the problem is reduced to proving that

$$V : X_{\mathrm{seq}} \to C_{\mathrm{seq}}, \quad \{D_n(f)\}_{n=0}^{\infty} \mapsto \{\lambda_{n+1}(c_{n+1}p_n - c_n p_{n+1})\}_{n=0}^{\infty},$$

is weakly compact.

The continuity of the operators $(U(\mu), \|\,\|_U) \to (U(\mu), \|\,\|_\infty)$, $f \mapsto f$, and $(U(\mu), \|\,\|_\infty) \to L^2(\mu)$, $f \mapsto f$, in conjunction with Plancherel's theorem yields the continuity of the operator

$$H : (U(\mu), \|\,\|_U) \to \ell^2, \quad f \mapsto \{c_n\}_{n=0}^{\infty}.$$

Moreover, since ℓ^2 is reflexive, the operator H is weakly compact (see [9, Proposition 3.5.4]). The assumed boundedness of $\{\|p_n\|_\infty\}_{n=0}^{\infty}$ implies that the operator

$$W : \ell^2 \to C_{\mathrm{seq}}, \quad \{\xi_n\}_{n=0}^{\infty} \mapsto \{\lambda_{n+1}(\xi_{n+1}p_n - \xi_n p_{n+1})\}_{n=0}^{\infty},$$

is well-defined and continuous. Since we may identify $U(\mu)$ with X_{seq}, the operator $V = W \circ H$ is a composition of a weakly compact operator and a continuous operator and therefore is weakly compact itself. ∎

The proof goes along the lines of the proof of [16, Theorem 4.1(b)]. We should mention that the functions given in the proof in [16] do not really separate the points, but this drawback can be eliminated easily as we have done in our proof here. The main reason why we can apply the ideas of [16] is Lemma 2.2 above. Also note that our results imply that if μ has property (B), then $U(\mu)^\star$ is a so-called separable distortion of an L^1-space (see [16, Section 3]).

4. The Dunford–Pettis property with respect to $U(\mu)$ and $U(\mu)^\star$. Another Banach space characteristic which attracts attention is the so-called Dunford–Pettis property. Note that a sequence $\{x_n\}$ in a normed space $(X, \|\,\|)$ is called *weakly null* if $\lim_{n\to\infty} x^\star x = 0$ for all $x^\star \in X^\star$, and *norm null* if $\lim_{n\to\infty} \|x_n\| = 0$.

Definition 4.1. Let X and Y denote two Banach spaces. An operator $T : X \to Y$ is said to be *completely continuous* if it takes weakly null sequences to norm null sequences. The Banach space X has the *Dunford–Pettis property* if every weakly compact operator $T : X \to Y$ is completely continuous.

There are other characterizations of the Dunford–Pettis property (see [9, 3.5.18]). A well-known Banach space with the Dunford–Pettis property is the Banach space $C(K)$ of scalar-valued continuous functions on a compact Hausdorff space K (see [6]). If the dual space $X^\star$ of a Banach space X has the Dunford–Pettis property, then so does X (see [9, Exercise 3.60]). By the Gelfand–Naimark theorem [5], $C(K)^{\star\star}$ is isometrically isomorphic to $C(\Sigma)$, where Σ denotes the maximal ideal space. Thus, as a simple consequence, $C(K)^\star$ and any higher dual space of $C(K)$ have the Dunford–Pettis property. Hence, if $U(\mu) = C(S)$, then $U(\mu)$, $U(\mu)^\star$ and any higher dual space have the Dunford–Pettis property.

Note that in the following the adjoint operator of an operator T is denoted by $T^\star$ and the adjoint operator of $T^\star$ is denoted by $T^{\star\star}$. In order to check whether an operator T is completely continuous, one can use the following lemma (see [16, Lemma 4.3]):

Lemma 4.2. *Suppose that* $(X, \| \|_X)$, $(Y, \| \|_Y)$ *and* $(Z, \| \|_Z)$ *are Banach spaces and let* $T : X \to Y$ *be a continuous linear operator. Further let* $W : X \to Z$ *be a weakly compact operator such that* $W^{\star\star}$ *is completely continuous. If* $\lim_{n\to\infty} \|Tx_n\|_Y = 0$ *for any bounded sequence* $\{x_n\} \subset X$ *with* $\lim_{n\to\infty} \|Wx_n\|_Z = 0$, *then* $T^{\star\star}$ *is completely continuous.*

Basing on the assumption that X is a closed subspace of $C(K)$, Cima and Timoney [3] used methods provided by Bourgain [1] to check whether X or $X^\star$ has the Dunford–Pettis property. The following lemma holds (see also [16]).

Lemma 4.3. *Let* K *be a compact Hausdorff space and let* X *be a closed subspace of* $C(K)$. *Set*

$$X_b = \{\gamma \in C(K) : T_\gamma \text{ is completely continuous}\}, \tag{4.1}$$

$$X_B = \{\gamma \in C(K) : T_\gamma^{\star\star} \text{ is completely continuous}\}. \tag{4.2}$$

Then X_b *and* X_B *are closed subalgebras of* $C(K)$. *Moreover, if* $X_b = C(K)$, *then* X *has the Dunford–Pettis property, and if* $X_B = C(K)$, *then* $X^\star$ *and* X *have the Dunford–Pettis property.*

The above X_b and X_B have been introduced in [3] and called *Bourgain algebras*. With regard to T_γ see Definition 3.2. Now, we are able to prove the following theorem.

THEOREM 4.4. *If μ has property* (B), *then $U(\mu)$ and $U(\mu)^\star$ have the Dunford–Pettis property.*

Proof. Again we refer to the notations and definitions of the proofs of Lemmas 1.1 and 2.2. As in the proof of Theorem 3.3 we set $X = J(U(\mu))$ and $X_{\mathrm{seq}} = J_1(U(\mu))$.

Let m be the positive measure on K defined by $m(\{1/n\} \times \mathcal{S}) = 0$ for all $n \in \mathbb{N}$ and $m|_{\{0\}\times\mathcal{S}} = \mu$. It can be shown easily that the natural embedding $\tilde{W} : C(K) \to L_1(K, m)$, $\varphi \mapsto \varphi$, is weakly compact. Thus, by Gantmacher's theorem [9, 3.5.13], $\tilde{W}^{\star\star}$ is weakly compact too. Since $C(K)^{\star\star}$ has the Dunford–Pettis property, $\tilde{W}^{\star\star}$ is completely continuous.

The restriction $W = \tilde{W}|_X$ is also weakly compact. Note that $X^{\star\star}$ is isometrically isomorphic to the closed subspace $(X^\perp)^\perp \subset C(K)^{\star\star}$ (see [9, 1.10.15, 1.10.16, 1.10.17]), where $^\perp$ denotes the annihilator. Hence, we may identify $W^{\star\star}$ with $\tilde{W}^{\star\star}|_{(X^\perp)^\perp}$, which implies $W^{\star\star}$ is completely continuous.

The operators T_{ϕ_k}, $k \in \mathbb{N}_0$, are bounded and of finite rank. Hence, they are compact (see [9, 3.4.3]). Therefore, by Schauder's theorem [9, 3.4.15] the operators $T_{\phi_k}^{\star\star}$, $k \in \mathbb{N}_0$, are compact too and by [9, 3.4.34] they are completely continuous.

Let $V : X_{\mathrm{seq}} \to C_{\mathrm{seq}}$, $\{D_n(f)\}_{n=0}^\infty \mapsto \{\lambda_{n+1}(c_{n+1}p_n - c_n p_{n+1})\}_{n=0}^\infty$. By property (B) we have

$$\begin{aligned}\|V(\{D_n(f)\}_{n=0}^\infty)\|_{C_{\mathrm{seq}}} &= \sup_{n\in\mathbb{N}_0} (|\lambda_{n+1}|\, \|c_{n+1}p_n - c_n p_{n+1}\|_\infty) \\ &\le 2 \sup_{n\in\mathbb{N}_0} |\lambda_{n+1}| \sup_{n\in\mathbb{N}_0} \|p_n\|_\infty \sup_{n\in\mathbb{N}_0} |c_n| \\ &\le M \int_{\mathcal{S}} |f|\,d\mu = M \int_K |J(f)|\,dm \quad \text{for all } f \in U(\mu).\end{aligned}$$

Therefore, we can apply Lemma 4.2, which implies that $T_{\phi_\infty}^{\star\star}$ is completely continuous. Finally, Lemma 4.3 and the Weierstrass theorem [7, (7.34)] imply $X_B = C(K)$, and so $X^\star$ and X have the Dunford–Pettis property. ∎

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