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Photoacoustic group velocity

C García-Segundo[1,2,*], F Filbir[2], E I Fuentes-Oliver[1] and V M Moock[1]

[1] Instituto de Ciencias aplicadas y Tecnología (ICAT), Universidad Nacional Autónoma de México, Circuito Exterior S/N, Ciudad Universitaria, 04510, Mexico City, Mexico
[2] Institute of Biomedical Imaging (IBMI), Helmholtz Zentrum München, Ingolstädter Landstraße 1, 85764 Neuherberg, Germany
[*] Author to whom any correspondence should be addressed.

E-mail: crescencio.garcia@icat.unam.mx

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Abstract

The concept of photoacoustic (PA) group velocity (v_g) is largely missed in the related transport theory. Generally it is assumed that the PA signal propagation occurs at a constant velocity, and thus overlooking the attenuation and dispersion effects. In practice, the PA signals propagate as material waves coupled to the sample, and thus inherently experiencing wave dispersion. In this work, such limitation is bypassed using a modified wave-function formulation in which both amplitude attenuation and wave dispersion arise naturally from a Heaviside-type (Cattaneo) transport equation operating on the velocity potential. Thus the approach, is valid for homogeneous media, characterised by a frequency-dependent PA absorption coefficient and an explicit expression for the group velocity. The resulting model accurately predicts the experimentally observed amplitude decay, bandwidth reduction, and peak-frequency shifts of PA spectra as functions of propagation distance. Furthermore, it is consistent with local thermal equilibrium and energy conservation requirements, and provides the necessary framework to satisfy Biot's group-velocity theorem and, consequently, the macroscopic Debye kinetic theory. This analysis also supports the interpretation of the PA signal as a dispersive wave packet propagating with an average group velocity, whose velocity distribution is governed by its spectral content.

1. Introduction

The interpretation of the photoacoustic (PA) velocity, both experimentally and theoretically, has been widely discussed due to its importance in the analysis and imaging of PA signals [1–6]. Although studies have explored ultrasound dispersion [7–13], a specific description of the velocity of a PA group, v_g, including its physical implications for PA signal interpretation and image formation, is lacking [14, 15]. This work addresses this gap by introducing the concept of PA group velocity. As discussed later, including this concept in the PA imaging framework enhances the physical information obtained when the PA signal propagates over distances that exceed those defined by the stress- and thermal-confinement conditions.

The formalism commonly used to describe the propagation of the generated PA signal is the transport theory, where the motion is modelled using a wave function derived after considerations of thermoelastic expansion for an incompressible fluid, and departing from the thermoelastic Navier–Stokes equation [12, 16, 17]. A more straightforward approach, adopted here, employs the velocity potential, $\phi(z, t)$ [18], because PA transport is irrotational; that is, the energy flux lacks rotational components, enabling the velocity to be defined as $v_z(z, t) = \partial_z\phi(z, t)$. Although the current analysis is limited to one dimension, the linearity assumption and material homogeneity allow for extension to higher dimensions, provided the sample under consideration by nature is a homogeneous medium. Therefore as initial condition is recognised that the PA signal generation and propagation fulfils energy balance conditions with local thermal equilibrium [19]. The introduction of the velocity potential as part of the formalism, has the benefit of portraying how is projected the velocity distribution at any point along PA propagation-pathway. This approach allows for the extraction of kinetic and material data from the PA velocity distribution, potentially enabling PA imaging based on velocity distribution, noting that Bernoulli's equation for pressure is $p + \frac{1}{2}\rho|\overrightarrow{v}|^2 = \text{constant}$, where the gravitational component is negligible in most practical cases.

Without loss of generality, we can set the constant to zero, resulting in $p(z,\ t) = -\frac{1}{2}\rho[\partial_z\phi(z,\ t)]^2$. This equation simply relates the PA pressure to the velocity potential, scaled by ρ, the mass density per unit volume. This formalism clarifies that the PA effect can sustain reversibility; that is, there is the inverse problem, and excess free energy can be treated as following a thermodynamic regime of local reversibility [18, 20–24].

Therefore, the thermodynamics of PA transport should be treated as a reversible process, consistent with the Green-Naghdi formalism and subsequent clarifications [25–28]. This avoids the non-physical infinite sound speed predicted by the Navier–Stokes equations, as discussed in detail in [29]. This approach allows us to represent the PA velocity as a spectral distribution, correcting the misconception of a constant PA velocity. Furthermore, it provides a framework to identify the group velocity, v_g, as the envelope's shape. General wave and transport theories support these properties, provided that the excitation energy does not cause phase changes or ablation. However, attenuation and acoustic dispersion, also known as spectral dispersion, modify the PA signal. Attenuation causes the amplitude to decay exponentially [11–14], while spectral dispersion contracts the spectral distribution, shifting the peak frequency to lower energies and altering the velocity distribution. This change becomes significant when the transport distance exceeds the PA source length, z_o, or when the spatial PA pulse-width is smaller than the transport distance. Thus, dispersion can be estimated from changes in the spectral distribution envelope, characterised by v_g. The spectral change increases with PA transport distance. It means consumption or dissipation of kinetic energy, in agreement with the energy conservation principle. As noted by Biot [7], there is an equivalence between v_g and the energy transport, stated by the v_g's theorem. This theorem states that, this velocity is defined by the energy flux across a fixed plane per unit time, $\partial_t H$, and divided by the total energy density, H_o. In this case, H_o is the initial PA energy defined by $H(t = \tau_L) = H_o$, where $\delta t = \tau_L$ is the time-width of a square-shaped laser pulse. In general, the effective transport time, $\Delta t = t_z$ is much larger than the laser pulse, $\tau_L \ll t_z$. Experiments typically show that for laser pulses with $\tau_L \simeq 7$ ns, the PA pulse time-width emerging from a sample is $\tau_{PA} \sim 1\ \mu$s, and this value depends on the transport distance and sound velocity, v_s. Thus the shortest transport time shall match the PA pulse-width. Although the current analysis focuses on biological tissue, where $v_s \sim 1500$ m/s is a reasonable approximation, the resulting framework can be generalised to other condensed media, [18].

2. Energy balance with non-equilibrium thermodynamics

To establish a physically consistent framework for introducing the PA-group-velocity, it is essential to first delineate the thermodynamic principles governing the signal's generation and transport. Conventional models often bypass the explicit thermodynamic sequence that connects optical absorption to the launch of a mechanical wave. The foundations for better understanding this problem are placed in section 2.1 with an analysis of the energy balance during the laser-matter interaction to define the initial PA energy density. At macroscale the thermodynamics is the non-equilibrium as described by Onsager and Maixner-Prigogine formalism. The details are described in the section 2.2. There we introduce the non-equilibrium considerations required to model the reversible conversion of this localized heat into a propagating stress wave, using the formalism of local thermal equilibrium, at finite velocity, overcoming the long-standing controversy. Together, these principles establish the thermodynamic legitimacy of treating the PA effect as a reversible process at the macroscale, setting the stage for the dispersive wave model developed in section 3.

2.1. Energy balance

Let $E_o(\overrightarrow{r})$ represent the energy of interaction. This analysis is limited to one dimension for simplicity, sufficient to introduce the key concept. The interaction occurs along the z-axis, with the boundary at $z = 0$ and $z > 0$ inside the sample. Under thermal equilibrium, the interaction is expressed by the energy balance between the flux of $E_o(\overrightarrow{r})$, the optical absorbance $\mathcal{A} = \alpha|\mathscr{E}_o(\overrightarrow{r})| = \alpha\mathscr{E}_o(z)$, and the photon emission, or emissivity, E_{ph}.

$$\frac{\partial}{\partial t}\mathscr{E}_o(\overrightarrow{r}) = -\partial_z q(z) + (E_{ph} - \mathcal{A}). \tag{1}$$

Assuming all absorbed energy is dissipated non-radiatively within the sample [18, 19], $E_{ph} = 0$. Since the laser pulse is described by the function $f(t) \triangleq \text{Rec}(t) = 1$ for $0 < t < \tau_L$ and zero otherwise, equation (1) simplifies as,

$$0 = -\partial_z q(z) - \alpha_o q(z)\ ; \tag{2}$$

which has the straightforward solution: $q(z = z_o) = q_o e^{-\alpha_o z_o}$.

This solution is the Beer–Lambert Law. For notational simplicity, sub-fixes are omitted with the understanding that their meaning is as defined. Here, q_o is the total initial energy flux acting as the source for the displacement potential. The initial PA energy density, H_o, is given by the energy conversion of non-radiative decays between the sample's boundary at $z = 0$ and the plane $z = z_o$, determined by the light's penetration depth, defined as the distance within the sample where the laser-pulse energy is attenuated below its half-width at full maximum (HWFM). This initial energy density is established within the time frame of the temporal laser pulse-width, which is orders of magnitude smaller than the PA temporal pulse-width, τ_{PA}. We assume $\tau_L = t_o \ll \tau_{PA}$ for the PA transport description at the macroscale, indicating that all optical absorption of energy, A, occurs within this time frame; $A = \alpha_o q_o e^{-\alpha z_o}$, with α_o being the optical absorption coefficient.

2.2. The local equilibrium in non-equilibrium thermodynamics

This thermodynamic framework uses non-equilibrium thermodynamics with the local equilibrium hypothesis. For small volumes at short timescales, such as τ_L and τ_{PA}, the intensive and extensive quantities are similar to those under reversibility. This is the well-known Onsager [22, 24] and Maixner-Prigogine formulation [24], supported by these assumptions: (a) local equilibrium applies; (b) the second law of thermodynamics holds; (c) linear constitutive equations (Fourier's law and other transport properties) can be used; and (d) the symmetry of Onsager's reciprocity relations is preserved. The last point is significant for ensuring the existence of the inverse problem and recovering the initial thermodynamic source. This framework also aligns with Lebowitz's [30] requirements for scaling thermodynamics from microscopic to macroscopic scales, placing the problem in a convex domain with analytic consequences. These consequences include thermodynamic stability, non-negative compressibility, and well-defined specific heat capacities, fulfilling the Dulong-Petit Law and allowing for the Debye formalism [24, 31].

At the macroscopic scale, the timescales τ_L and τ_{PA} are much shorter compared to the PA transport time, t_z, defined as the signal travel time from the PA source to the output boundary. The energy A is treated as thermal energy, triggering a distribution of mechanical resonators with frequencies characteristic of the medium and rated by the heat capacity, as in the Debye formalism. Since no chemical potential is involved, the local dynamics with local equilibrium imply that the free energy $F = U - TS$ is that of a reversible process, where U, T, and S are the local internal energy, temperature, and entropy, respectively. Reversibility is established as $F(z, t^-) = F(z, t > t_o)$, where t^- is any time before the laser pulse interaction. Thus, the net change of local free energy is,

$$\delta F = \delta(U - TS) = 0\,. \tag{3}$$

Instead, at $0 < t < \tau_{PA}$, the absorbed energy diffuses into the sample, consistent with Fourier's law and the PA transport expressed by the PA wave equation. The locally absorbed energy is transferred as heat, causing a local increase in entropy S. Such that,

$$T\frac{dS}{dt} = -\partial_z q(z) + \alpha_o q_o e^{-\alpha_o z_o} f(t); \tag{4}$$

where $\partial_z q(z)$ represents the divergence of the heat flux along the z-axis, with units of $Jm^{-2}s^{-1}$ in MKS units. More details on optical, thermal, and mechanical energy conversion can be found in [32].

Combining equations (3) and (4) shows that the main channel for the dissipation of the free energy excess is as heat, $\delta U = T\delta S$, prompting a burst of combined mechanical stress and strain, described by a displacement or velocity potential, as shown later. The thermodynamic treatment is that of energy transport in an elastic material as a reversible process. This problem has been widely discussed, though somewhat apart from PA theory, in the Green-Naghdi formalism [25] and further discussions [26–28], correcting the misinterpretation that leads to an apparent infinite sound velocity, comprehensively discussed in [29].

Reversibility and local energy balance ensure that for $t \leqslant 0$ and $t > t_z$, the initial and final times, the free energy and entropy are the same, $F = F_o$ and $S = S_o$, respectively. However, during $0 < t < t_z$, the free energy and entropy change as $F(T) = F_o + \Delta F(T)$, and $S(T) = S_o + \Delta S(T)$. Using the thermodynamic relationship $S(T) = -\frac{\partial F(T)}{\partial T}$ [18, 20, 21, 24], with

$$F(T) = F_0 - \mathbb{K}\beta(T - T_0)u_{ll} + \mu\left(u_{ik} - \frac{1}{3}\delta_{ik}u_{ll}\right)^2 + \frac{1}{2}\mathbb{K}u_{ll}, \tag{5}$$

the explicit description of $\Delta F(T)$ and $\Delta S(T)$ can be obtained.

In equation (5), u_{ll} is the trace of the strain (displacement) tensor, and δ_{ij} is the Kronecker delta. The non-diagonal terms in the strain tensor are defined as $u_{ik} = \frac{1}{2}(\partial_i u_k + \partial_k u_i)$, expressed in terms of displacement field components, u_i. Additionally, equation (5) includes the bulk modulus, $\mathbb{K} \triangleq \lambda + \frac{2}{3}\mu$, where λ and μ are the first and second Lamé coefficients, with μ also known as the shear stress modulus, and β is the thermal expansion coefficient.

Then the entropy change turns out to be, $S = S_o + \mathbb{K}\beta u_{ll}$. Since the PA field is oscillatory, as described by Hooke's law, the one-dimensional pressure field is written as, $p(z, t) = -\mathbb{K}u_{ll}$. With the trace of the strain tensor defined as $u_{ll} = \partial F/\partial \sigma_{ll}$, one can recognise that the light-induced thermal expansion is free from external forces. This means the induced strain, u_{ll}, is free from internal stress and therefore $\sigma_{ik} = 0$.

With the initial PA conditions established, the next step is to describe the transport process. Approaches focus on deriving a Helmholtz-like PA wave equation, either from the Navier–Stokes equation [15–17, 33], or from energy balance and solving a Green's function problem, or using thermodynamics with local equilibrium and conservation of momentum [18, 19, 34]. The thermodynamic stage is similar to that introduced by Green-Naghdi [25], accounting for the in-depth discussion in [29]. The Green-Naghdi formulation is fundamental in depicting the necessary and sufficient conditions to implement a Heaviside or Cattaneo-type differential equation. This approach details how energy flow should be interpreted as: heat conduction only, as a wave (the PA case), or propagation in a more general viscoelastic fluid. These cases are branded here as heat flow GN-I, GN-II, and GN-III, corresponding to type I, type II, and type III introduced by Green and Naghdi. The Green-Naghdi (GN) interpretation is founded on well-established postulates or axiomatic conditions, from which constitutive equations and expressions for energy propagation are built. Further, resulting expressions in the GN interpretation of energy flux can be scaled up or down: from a Helmholtz wave function derived from a Laplace-type equation, to the Cattaneo- or Heaviside-type equation, or the Jeffreys-type, in agreement with the conclusions in [26, 27, 29].

Those considerations are valid for the transport across the homogeneous sample, as is the current framework; details can be found in [1, 4, 5]. Even in cases where the target is changing position in time, this view is yet valid whenever the PA transport is free from physical and chemical changes of the sample's properties [35]. Instead, the factor to account for is the transport distance, in comparison with the dimensions of the thermal-confinement and the stress-confinement [36]. The comparison reveals the dimensions at which the propagation of PA signal, posed as a material wave package, can be sensitive to wave attenuation and wave dispersion; in similar fashion as treated in effective-medium wave theory [14, 37].

3. The extended PA transport model

In conventional PA transport, the propagation velocity is the phase velocity, v_s, implicitly assumed as the average value of a constant velocity distribution. If this were true, the spectral distribution, and thus the energy/information content, would remain constant regardless of propagation distance. However, experiments show spectral dispersion with consequent information loss when the propagation distance is much larger than the dimensionality defined by the thermal confinement condition [3, 4]. This approximation may suffice for PA sensing applications. As shown in [14], increasing sample density leads to information loss as the spectral distribution shifts to lower frequencies.

For interpreting the flux of PA energy, $\partial_t H$, we follow the analysis introduced in [32]. A dimensional analysis leads to the following definition for the energy flux per unit time and area, as: $\partial_t H = \alpha_o \kappa T$. Recalling that equation (4) assumes all absorbed optical energy is locally transferred to heat, then locally, $\partial_t \mathcal{H} = T\frac{dS}{dt} = \frac{dQ}{dt}$. As a result of the displacement, the induced momentum is $\rho \nabla \phi$, with the scalar velocity along each spatial j-component expressed in terms of the displacement field as $v_j = \partial_t u_j$, or simply $v = \partial_t u$ in the one-dimensional case.

Regarding PA transport distance, recall the distance-scaled dimensionless definition in terms of the Fourier number $\mathcal{D} = \sigma t_o/z_L^2$, where σ is the thermal diffusivity, defined as the ratio between heat conductivity, κ, and specific heat; thus $\sigma = \kappa/\rho^2 c$ having assumed $c = C/\rho$, with C the heat capacity.

PA transport is a causal process with initial and boundary conditions specific to each case. However, once defined, these conditions prevail. In addition to these conditions, this analysis highlights the necessity of including attenuation and spectral dispersion as necessary initial conditions. These additional conditions do not alter the expected result or computational cost if the problem does not improve upon their consideration. The significance of their inclusion improves information recovery and PA image reconstruction quality [4, 5, 14], in contrast to the most commonly used PA transport framework. It is customary to assume PA attenuation and spectral dispersion are negligible, without considering physical conditions to establish the maximum energy for reversibility to prevail. This issue is addressed by considering Debye theory for specific heat, satisfying the Dulong-Petit law. Its consideration in PA transport highlights the dependence of the frequency distribution (velocity distribution) on PA transport distance. Like any field interaction, PA propagation requires interaction of its mechanical field with the sample, resulting in work, energy dissipation, and PA field decay following an exponential law [7, 11–13].

Let PA transport be expressed by the linear transformation:

$$p(z,\, t) = \mathcal{L} p_o(z_o,\, t), \tag{6}$$

where $p(z_o) f'(t) \triangleq p_o(z_o,\, t)$ is the initial pressure within the PA source's extent, $0 \leqslant z \leqslant z_o$. After propagation, the measured pressure at the output boundary is $p(z, t)$, the PA signal. Without attenuation and spectral dispersion, $\mathcal{L} = \partial_z^2 - \frac{1}{v_s^2}\partial_t^2$, producing the Helmholtz-type PA wave equation with v_s (the constant sound velocity) as the PA transport velocity. With attenuation and spectral dispersion, analogy with wave theory [38] suggests that the wave equation should be: $\mathcal{L} = \partial_z^2 - \frac{1}{v_s^2}(-\partial_t^2 + \hat{a}(\omega)\partial_t + \hat{d}(\omega))$. In thermodynamics, this is known as a Cattaneo-type equation [25, 29], also recognised as a Heaviside differential equation. In wave theory, related to material waves, this is known as the telegrapher's equation or the Kelvin-Voigt equation deduced from the Navier–Stokes equation when viscosity is included; where the coefficients, $\hat{a}(\omega)$ and $\hat{d}(\omega)$ relate to PA diffusive wave dispersion and acoustic attenuation, respectively. In the frequency domain, the one-dimensional differential equation is,

$$\left\{\partial_z^2 + \frac{1}{v_s^2}[\omega^2 - i\omega\hat{a} - \hat{d}(\omega)]\right\}\hat{p}_o(z,\, \omega) = \hat{\mathcal{L}}\hat{p}_o(z,\, \omega) = \hat{p}(z,\, \omega). \tag{7}$$

Here $\hat{\dots}$ notation, thereafter, is used to indicate that the involved function is in the frequency domain.

This representation reduces the analysis complexity and is simpler to manipulate. In this representation the wave-dispersion and attenuation coefficients are, $\hat{a}(\omega)$ and $\hat{d}(\omega)$, respectively. The consistency conditions impose that these coefficients are frequency dependent, along with constant material properties, as shown in the remainder of this manuscript. These properties determine the initial and transport conditions influencing the transfer function, that is solution to the Ec. (7).

In the time domain, $p_o(z, t)$ is the response to the impulse function; in the frequency domain, it is the transfer function, $\hat{p}_o(z,\, \omega)$ or simply $\hat{p}_o(\omega)$ to indicate no spatial dependence. However, with PA attenuation and acoustic dispersion, the PA signal propagates with losses, so the sound velocity, v_g, (in Biot's interpretation as energy flux per unit time, $\partial_t H_z$, across a fixed plane at $z > z_o$, divided by the total energy density at that plane, H_z) differs from the initial value, $v_g = \frac{\partial_t H_{t_o}}{H_o}$. This arises from PA losses due to PA-field interaction with the media (friction-like or PA impedance [32]). Using a constant sound velocity, v_s, is only valid in media with hydrostatic compression, neglecting any mechanical resistance (viscosity, friction, or mechanical impedance to shear stress). In those conditions, the term in equation (5) containing μ can be neglected, simplifying the equation and resulting in the Helmholtz-type PA wave function, implying a constant PA velocity, v_s. This velocity can be defined in terms of static material parameters or constant average wave propagation parameters, as $v_s^2 = \mathbb{K}/\rho$ and $v_s = \overline{\omega}/\overline{K}$, respectively. The over-bar notation, $\overline{(\dots)}$, indicates constant frequency and wave number values; i.e free from considerations of attenuation and wave dispersion, and with the PA perturbation assumed as monochromatic.

Alternatively, in the definition related to material properties, where $\mathbb{K}$ is defined in terms of the first (λ) and second (μ) Lamé coefficients, $\mathbb{K} \triangleq \lambda + \frac{2}{3}\mu$, again, without shear stress, μ can be neglected, turning out that $v_s = \lambda/\rho$ is a constant. The PA transport is usually interpreted through the Helmholtz-type wave function.

3.1. PA group velocity

A more realistic and consistent interpretation of the PA transport phenomenon has to include the velocity distribution, exhibiting the true nature of the mechanical energy burst or wave packet, as bounded to the material and with the velocity distribution required to change with the propagation distance.

Such considerations, are also extensive to the inverse problem, and thus display conditions to ideally recover the initial PA signal, in the usual way,

$$p_o(z,\, t) = \mathcal{L}^{-1} p(z,\, t); \tag{8}$$

minding that the wave number evolves over the transport, due to the expected attenuation and dispersion of wave package made of velocity components in the range $\delta v(\omega)$. Thus, over the propagation

$$\delta v(\omega) = \delta\left(\frac{\omega}{\hat{K}(\omega)}\right) = \frac{\hat{K(\omega)}\,\delta\omega \ - \omega\,\delta\hat{K}(\omega)}{K^{\hat{2}}(\omega)}. \tag{9}$$

At the limit, the velocity, as function of the frequency, turns out to be $\delta v(\omega) \Rightarrow \frac{\partial\omega}{\partial\hat{K}(\omega)} \triangleq v_g(\omega)$; in agreement with the spectral dispersion wave theory [11, 13, 38]. Since the instant propagation velocity is independent of spatial parameters, the orthogonality between components allows for a partially perturbative treatment. Thus the current description can be generalized from the one-dimensional solution to a larger number of orthogonal components.

In this representation, $p_o(z, t)$ in equation (6) is the impulse response, and its Fourier representation it is the transfer function [11, 39]:

$$\hat{p}(z,\omega)=\hat{p}_o e^{-i\hat{K}(\omega)z}. \tag{10}$$

For the corresponding transport problem, with attenuation, the general consensus is that the transfer function maintains its form as in equation (10), following an exponential decay-law, analogous to Lambert-Beer law for optical attenuation [1, 2, 11, 12]. This is achieved by modifying the wave number to be a complex-valued function, with constant acoustic attenuation, α_A, as the imaginary term:

$$\hat{K}(\omega)=\frac{\omega}{v_s}+i\alpha_A(\omega). \tag{11}$$

Generalising this problem to include spectral dispersion in PA theory involves considering the velocity in (11) as a function of frequency, details can be found in [11]. Thus, the rate of change arises from its first derivative.

$$\frac{d\hat{K}}{d\omega}=\frac{1}{v_g(\omega)}+i\frac{d\alpha_A(\omega)}{d\omega} \tag{12}$$

An analogous result is obtained when attenuation and dispersion are initial terms in the wave function. In that case, the corresponding wave equation is the telegrapher's or Heaviside-type. To circumvent issues in [29], the thermodynamic framework, as described in the section of thermodynamic considerations, must be carried out through the GN interpretation [7, 12, 25–28, 40]. The benefit of this interpretation is shown in preliminary advancements [4, 5] for the continuum case, and in [14] for granular/colloidal media. The wave number in equation (10), is a complex function of the frequency;

$$\hat{K}(\omega)=\frac{\omega}{v_s}-\frac{1}{2}\omega^3 d+i\frac{1}{2}\omega^2 a, \tag{13}$$

whose first-order frequency derivative is,

$$\frac{d\hat{K}(\omega)}{d\omega}=\frac{1}{v_s}-\frac{3}{2}\omega^2 d+i\omega a. \tag{14}$$

Here $\hat{a}(\omega)=\omega a$ and $\hat{d}(\omega)=\frac{3}{2}\omega^2 d$. The comparative analysis of outcomes posed by equations (11) to (14), permit to obtain explicit relationships for $\mathtt{a}$ and $\mathtt{d}$, in term of actual physical properties, and thus to better understand the proper nature and pertinence of consideration of the PA's attenuation and spectral dispersion. To better understand these terms, next we place a discussion about its actual physical meaning and dependence with the physical properties of the sample.

3.2. PA attenuation

By comparing terms between equations (11) and (13), the complex part equates to,

$$\alpha_{_{PA}}(\omega)=a\omega^2. \tag{15}$$

This result is analogous to that in [12]-§79. The correspondence between parameters suggests that the a coefficient in (19) corresponds to a in equation (79.6), and since $H_o(z_o,\omega)=\frac{1}{2}\rho v_s^2$, then

$$a=\frac{1}{v_s\mathcal{H}_o}\left[\left(\frac{4}{3}\eta+\zeta\right)+\kappa\left(\frac{1}{c_V}-\frac{1}{c_p}\right)\right]. \tag{16}$$

Here η is the dynamic viscosity and in the fluids interpretation it corresponds to the Lamé μ coefficient; consequently η corresponds to λ's Lamé coefficient. The change of representation is dialectic in nature. In the current context, it is more convenient to treat the PA propagation, like a displacement potential. It means to assume related energy to be transported in the same manner as a fluid, travelling across in a closed perturbed way, and at sound speed. In addition, by using the adiabatic index or Laplace's coefficient, defined as $\gamma = c_p/c_V$, and $\kappa = \sigma\rho c_V$; thus equation (16) is rewritten as,

$$a=\frac{1}{v_s\mathcal{H}_o}\left[\left(\frac{4}{3}\eta+\zeta\right)+\sigma\rho\left(1-\frac{1}{\gamma}\right)\right]. \tag{17}$$

It means that there are two pathways for PA attenuation. One is the inertial loss of energy, as indicated by the term with the Lamé coefficients; analogous to fluid's viscosity or mechanical impedance; that is constant at any direction of propagation. The other pathway the dissipation of energy in diffusive way, as part of the coupling of the PA material burst and thus be transported, in effective terms, as a wave-package.

Since biological tissue can be assumed as a colloid, those examples are a good enough approximation for explanatory purposes. Once a sample is given, the volume fraction of the granularity, say inhomogeneities, it is constant, and so it is the corresponding rate of inertia. In this scheme the term with the Lamé parameters pose a constant value; because this kind of attenuation appears to be uniform for any direction of PA propagations. Instead the diffusive terms relates to cumulative losses that increase with the propagation distance.

Since $\mathcal{H}_o(z_o, \omega) = \frac{1}{2}\rho v_s^2$, then the a coefficient can be approximated as,

$$a = \frac{2\sigma\rho}{v_s^3}\left(1 - \frac{1}{\gamma}\right). \tag{18}$$

Therefore the PA attenuation coefficient ought to be,

$$\alpha_{PA}(\omega) = \frac{2\sigma\rho}{v_s^3}\left(1 - \frac{1}{\gamma}\right)\omega^2. \tag{19}$$

The term related to viscoelastic contributions accounts, in the more frequent cases, represent a constant value. The case for aeroelasticity without diffusivity requires additional considerations as stated in [25]. It poses a problem out of the current scope and therefore, at the moment, is left out of discussion. Interested readers can consult for instance, [12]:§81.

Given the above result for the complex term in equation (13), and accounting for the corresponding considerations [12, 13], one would expect that the Lambert-Beer-like equation for PA attenuation would follow the exponentially decaying law,

$$\mathcal{A}_{PA}(z, \omega) = \mathcal{H}_o e^{-\alpha_{PA}(\omega) z} = \mathcal{H}_o e^{-a\omega^2 z}. \tag{20}$$

The structural inhomogeneities cause to increase the effective path way of propagation. At macroscopic scale this is observed as changes in the propagation velocity. Since the PA transport is a causal phenomenon, in the same sense as temperature, the instantaneous PA transport pathway cannot be known. Any observed PA signal is in fact an integrated average, whose shape and time width depends upon the propagation distance, and thus the loss of kinetic energy. This is a practical representation of the equation (9), alas in the Fourier space. That is, the frequency bandwidth keeps shrinking with the distance, whilst the frequency peak value keeps shifting to low frequencies [4]. This process is simultaneous with the signal attenuation. Wave theory shows us that this phenomenology corresponds to the spectral dispersion, as described in wave theory. More specifically, from equation (20), is apparent that this means dissipation of kinetic energy; and therefore the velocity distribution is due to be modified, as discussed next.

3.3. PA dispersion

From the equations (11) to (14), is evident that the spectral dispersion is related to the real part of the wave number. In particular, from its first derivative, given by (12) and (14); which then shall equate as,

$$\frac{1}{v_g} = \frac{1}{v_s} - \frac{3}{2}\omega^2 d; \tag{21}$$

Since the Biot's theorem provides an explicit way of defining the group velocity, then to get an explicit expression for d is a straightforward task, which is described as follows.

The Biot'st theorem states that the group velocity is given by the quotient between the energy flux, $\partial_t H(\omega)$, and the $\mathcal{H}_o(z_o, \omega)$.

The energy flux for the PA phenomena is rated per unit time, unit area and unit temperature, as noted in equation (4). This can be obtained from equation (79.3) in [12]$-$§79, and by Eq, (16), in [32], which is used next in view of its simplicity.

$$\partial_t \mathcal{H}(\omega) = \alpha_o \kappa T \tag{22}$$

where minus indicates loss of energy. When divided by the total energy density, one gets the group velocity expressed as,

$$v_g(\omega) \triangleq \frac{\partial_t \mathcal{H}(\omega)}{\mathcal{H}_o(z_o, \omega)} = \frac{2\alpha_o \kappa T}{\rho v_s^2}. \tag{23}$$

Then the coefficient d, from (21), is $d = \frac{2}{3}(\frac{1}{v_s} - \frac{1}{v_g})\omega^{-2}$; or more explicitly,

$$d = \frac{2}{3}\left(\frac{1}{v_s} - \frac{\rho v_s}{2\alpha_o \kappa T}\right)\frac{1}{\omega^2} \tag{24}$$

For closing this subsection, last to recall equation (3), which, combined with its relationship to the specific heat capacity, then

$$c_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{\partial F}{\partial T} + T\frac{\partial S}{\partial T} = -\mathbb{K}\beta u_{ll} \tag{25}$$

This indicates that the net change in internal energy, is graded by the specific heat capacity, that equates with the total absorbed energy, which in turn is totally transferred to the displacement potential. In the statistical interpretation, at granular micro scale, this corresponds to perturbation of the vibrational states or vibration modes, distributed in the range $(\omega, \omega + d\omega)$.

In the Debye model interpretation of the heat capacity, the above interpretation implies that the total absorbed optical energy, is converted to transport energy expressed by through N-oscillation modes, in total agreement with the Dulong-Petit law. These oscillations propagate across the transport distance, in diffusive mode and fulfilling the local equilibrium conditions needed in the Maixner-Prigogine formulation to satisfy the local reversibility condition. In the Debye interpretation, the heat capacity is defined as,

$$c_V = \left(\frac{\partial U}{\partial T}\right) = \frac{z\hbar}{2\pi^2 v_s^2}\int_0^{w_D}\frac{w^3}{e^{\frac{\hbar w}{k_B T}} - 1}dw \;\Rightarrow\; c_V \simeq Nk_B T \equiv 2\pi h N\omega; \tag{26}$$

The integral has the noted approximate solution, in terms of an ideal gas; where k_B is the Boltzmann constant. We recall from Deby's kinetic theory Then after accounting for $\kappa = \sigma\rho C_j$; with c_j the heat capacities, which at macroscale the difference is such that they can be interchanged without affecting the actual meaning of the end result. Then

$$v_g = \frac{2\alpha_o\sigma T}{v_s^2}c_V = \frac{\alpha_o\sigma}{k_B v_s^2}8\pi^2 N\omega^2; \tag{27}$$

Since equation (24) can be written in terms of the group velocity as:

$$\omega^2 d = \frac{2}{3}\left(\frac{1}{v_s} - \frac{k_B v_s^2}{8\pi^2\alpha_o\sigma N\omega^2}\right); \tag{28}$$

then the real part of the wave number result to be purely dependent on the group velocity, v_g, and thus of the frequency, as

$$\hat{K}_{re}(\omega) = \frac{k_B v_s^2}{8\pi^2\alpha_o\sigma N\omega^2}. \tag{29}$$

The experimental conditions for wave attenuation depend on the number or density of resonators per unit volume, along with their spectral frequency distribution (i.e., the spectral density distribution or absorption spectrum). This implies that a portion of the absorbed energy is governed by diffusive attenuation, leading to losses in the PA kinetic energy. These implications become apparent when expressing the PA pressure through the velocity distribution.

As noted earlier, a linear relationship exists between the group velocity and the frequency. Without loss of generality, the spectral distribution is better expressed in terms of the invers introducing the velocity potential as part of the formalism, and thus, project velocity distribution at any point along PA prope lattice number, dn; which only depends on the oscillation frequency. Then, n is the particle density proportional to the frequency density, and then

$$D(\omega) = \frac{dn}{d\omega} \sim \frac{1}{8\pi^2}\frac{\alpha_o\sigma}{k_B v_s^2}N\omega^2, \tag{30}$$

is the frequency distribution density. By acknowledging the uncertainty principle as inherent in the determinism governing PA phenomena, then, it turns out that in practice, it is impossible to measure the instantaneous PA pressure. Any measured signal is an average determined by the time constant of the sensing device. Thus, the spectral dispersion imposes that the wave number, although pointing in the same direction, would exhibit changes with the propagation distance.

In practical applications, means that the content of information in any PA output, it would depend on the direction of observation. For instance, in biological tissue, a given sample may look inhomogeneous, and however in average the degree of inhomogeneity, say volume fraction, is the same in all directions. However, as the PA propagates, the losses leading to attenuation and spectral dispersion keep accumulating with the distance of propagation. Since the PA transport distance, changes with the direction of observation, then also would be the rate of the attenuation and spectral dispersion. This phenomenology is a natural of any material sample, as consequence that the transport is inherently bounded to the matter. Thus it should not be confused with a sort of anisotropy.

This discussion is closed highlighting that the transfer function described by equation (10), has to be modified in view of the resulting complex valued wave number,

$$\hat{K}(\omega) = \hat{K}_{re}(\omega) + i\frac{1}{2}\omega^2 a. \tag{31}$$

Being its real part represents the PA dispersion, and the complex part relates to the amplitude attenuation, both dependent on the PA frequency and the transport length; the resulting transfer function ought to be,

$$\hat{p}(z,\,\omega) = \hat{p}_{_{PA}}\, e^{-i\hat{K}_{re}(\omega)z}; \tag{32}$$

where $\hat{p}_{_{PA}} = \hat{p}_{o} \exp(-2\alpha_{PA} z)$.

This expression is quite general, since cover conditions at which the PA spectral dispersion is negligible. In those cases, one can assume $\mathtt{d} = 0$, recovering the transfer function as in [1, 2]. Also, on the assumption that the PA transport is free from attenuation and dispersion, as is styled in the foremost common interpretation, then $\mathtt{a} = 0$ and $\mathtt{d} = 0$, recovering the conventional transfer function, as described by the Helmholtz wave equation. Also, the explicit expressions for $\mathtt{a}$ and $\mathtt{d}$, give the handle to envisage the dominant mechanisms involved in the attenuation and dispersion. The assessment for the transport distance and physical properties for which these coefficients can be included or ignored, is at hand, since all these parameters can be determined either from the experiment or from material properties listed in handbooks or any other reliable source.

4. Conclusions

We introduced a physically consistent framework for photoacoustic (PA) transport that incorporates both attenuation and spectral dispersion, revealing the necessity of defining a PA group velocity, v_g. The usual PA theory assumes propagation at a constant velocity, v_s, and neglects dispersive effects. Such approximation is only valid for transport distance that are much smaller than the characteristic dimensions established by the stress- and thermal-confinement [36]. By adopting a thermodynamic formulation based on local equilibrium and reversibility–consistent with the Green-Naghdi [25, 26] theory, the PA field is shown to propagate as a dispersive material wave, whose transport evolution is governed by a Heaviside– (Cattaneo)-type equation.

Within this framework, one attains a representation of the PA signal transport closer to the experimental observations reported elsewhere in the literature. In addition it shows in a clearer manner that the PA signal is better interpreted as a wave packet with a frequency-dependent velocity distribution. Attenuation arises from viscous and diffusive mechanisms, while spectral dispersion contracts the frequency bandwidth and shifts the spectral peak toward lower frequencies. These effects become increasingly significant with propagation distance and are quantitatively described by frequency-dependent attenuation and dispersion coefficients, derived from material parameters such as thermal diffusivity, heat capacity, and the Lamé coefficients.

The group velocity naturally emerges as the ratio of PA energy flux to energy density, in accordance with Biot's theorem. This velocity reduces with distance due to cumulative dissipative processes and directly reflects the redistribution of vibrational energy among the medium's resonant modes. The resulting interpretation aligns with the Debye model at the Dulong–Petit limit, highlighting that absorbed optical energy is transferred into a spectrum of mechanical oscillators, following the detailed balance and thermal equilibrium at the micro scale, and then transferred as mechanical vibrations whose density and frequency distribution determine the actual effective propagation velocity, v_g.

The resulting dispersive transfer function incorporates both real and imaginary components of the complex wave number, thereby generalizing the Helmholtz equation commonly used in PA modelling. This formulation unifies PA attenuation, spectral dispersion, and velocity distribution under a single thermodynamically consistent model. It also clarifies the conditions under which the simplified formalism, neglecting attenuation or dispersion, or both, may still be applied.

Overall, this study provides a rigorous physical basis for interpreting PA transport in terms of group velocity and dispersive wave propagation. We believe that this representation brings accuracy to the PA signal analysis and improve conditions for better quality image reconstruction, particularly in strongly attenuating media. This approach is neither stringent nor depending on chance, instead offers explicit criteria for determining when dispersion and attenuation must be included in PA modelling, relying explicitly in physical material properties.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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